Construction of a Two-Dimensional Finite Difference Time Domain Method Based on a Simplex Grid

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Abstract — A 2-D finite difference time domain (FDTD) method based on a simplex grid for electromagnetic scattering is presented in this paper. Because of the scattering objects with realistic and irregular shapes, the method is based on a triangle and hexagon grid which is different from the rectangle grid in the traditional FDTD method. And it is especially the specifying incident source and the absorbing boundary condition using the triangle grids. The proposed method models objects with realistic and irregular shapes encountered in practical electromagnetic scattering problems.

I. INTRODUCTION

Numerical modeling has been widely accepted as an efficient tool for the accurate solution of a great variety of electromagnetic problems [1]–[7]. When analyzing the transient problem, MOM (Moments of Methods) and Finite Element Methods (FEM) must repeat the same calculation at each frequency point. Finite Difference Time Domain (FDTD) method can be a very useful tool to deal with transient problem. But for electromagnetic problems having objects with irregular shapes, the direct and fine discretization which is used to model the shape of the objects precisely may require a very dense mesh in which in turn requires a very heavy computational burden. In this paper, a 2-D finite difference time domain method based on a simplex grid will be presented. The method is based on a triangle grid which is different from the rectangle grid used in the traditional FDTD method.

II. THE CONSTRUCTION OF THE METHOD

Most two-dimensional problems can be decomposed into separate problems, each including separate field components that are transverse electric and transverse magnetic fields for the case under consideration. In this paper the transverse electric (TE) field is taken as an illustrating example. The equation of the transverse electric field is as shown in (1)-(3):

$$\nabla\times\eta_0\left(\vec{n}H_n+\vec{s}H_s\right)=\vec{z}\,\eta_0\sigma E_z+\vec{z}\,\varepsilon_r\frac{\partial E_z}{\partial t} \quad (1)$$

$$\frac{\partial E_z}{\partial n}=\rho H_s+\mu_r\eta_0\frac{\partial H_s}{c\partial t} \quad (2)$$

$$\frac{\partial E_z}{\partial s}=-\rho H_n-\mu_r\eta_0\frac{\partial H_n}{c\partial t} \quad (3)$$

Using Stokes's Formula, (1) can be changed into the (4):

$$\oint_S \eta_0 H_s ds=\iint_A\left(\eta_0\sigma E_z+\varepsilon_r\frac{\partial E_z}{\partial t}\right)dA \quad (4)$$

The FDTD updating equations based on the simplex grid for the TE case can be obtained by applying the central difference formula to the equations constituting the TE case based on the field position as shown in following Figures. The FDTD updating equations for the TE case are obtained as (5) and (6):

$$\sum_{k=1}^{6}\eta_0 H_{s,[k,j]}^{m+\frac{1}{2}}l_{[k,j]}=\eta_0\sigma A\frac{E_{z,[j]}^{m+1}+E_{z,[j]}^{m}}{2}+\varepsilon_r A\frac{E_{z,[j]}^{m+1}-E_{z,[j]}^{m}}{c\Delta t} \quad (5)$$

$$\frac{i}{d[k,j]}\left(E_{z,[k]}^{m}-E_{z,[j]}^{m}\right)=\rho\frac{H_{s,[k,j]}^{m+\frac{1}{2}}+H_{s,[k,j]}^{m-\frac{1}{2}}}{2}+\mu_r\eta_0\frac{H_{s,[k,j]}^{m+\frac{1}{2}}-H_{s,[k,j]}^{m-\frac{1}{2}}}{c\Delta t} \quad (6)$$

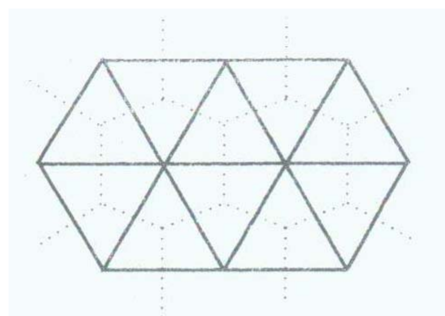

Fig. 1

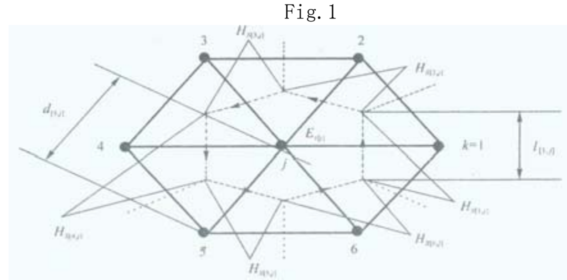

Fig. 2

III. EXAMPLE

This example introduces two-dimensional simulation. It begin with the basic two-dimensional formulation in FDTD and a simple example using a point source.

The program implements the above equations. The numerical programme of the presented method will be described in details in full paper. It has simple Gaussian pulse source that is generated in the middle of the problem space. The Fig.3 ~Fig.6 demonstrates a simulation for the first 50 time steps.

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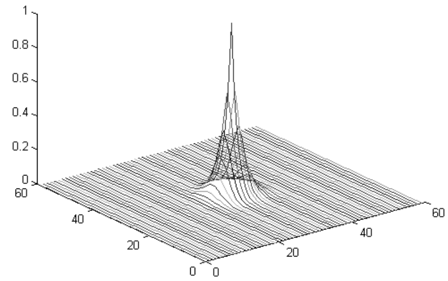

Fig. 3

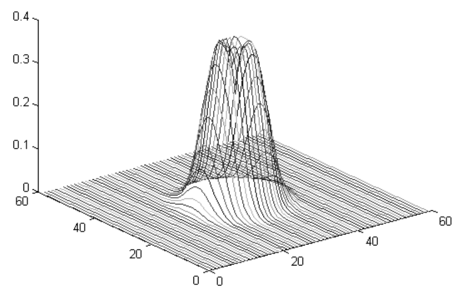

Fig. 4

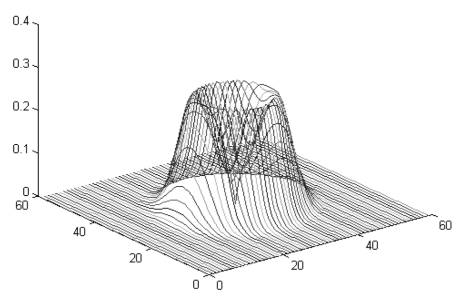

Fig. 5

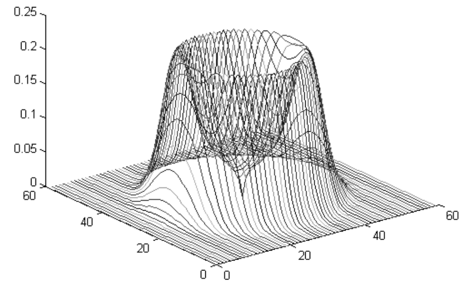

Fig. 5

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